

A Model Of Inflationary Cosmology Without Singularity

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In this letter, we propose a model of inflationary cosmology with a bounce preceded and study its primordial curvature perturbations. Our model gives rise to a primordial power spectrum with a feature of oscillation on large scales compared with the nearly scale-invariant spectrum generated by the traditional slow rolling inflation model. We will show this effect changes the Cosmic Microwave Background (CMB) temperature power spectrum and the Large Scale Structure (LSS) matter power spectrum. And further with a detailed simulation we will point out this signal is detectable to the forthcoming observations, such as PLANCK and LAMOST.

Inflation, as a description of the very early universe, has successfully resolved some problems existing in hot Big Bang cosmology, such as flatness, horizon, monopole problem and so on [1]. However, this scenario is puzzled by the initial singularity [2]. One possible approach to this disaster is to introduce a bounce before the inflationary expansion, which requires the hot Big Bang expansion be preceded by a contracting period [3, 4, 5]. If this happens, one significant question would be proposed: what does bouncing cosmology tell us for observations? Or, is it detectable for some primordial relics from contracting phase to be imprinted on observations? To answer this question, we need to study the evolution of primordial gravitational perturbations seeded before the bounce.

In this letter, we propose a nonsingular inflationary model, and study signatures of its primordial fluctuations on CMB temperature power spectrum and LSS matter power spectrum. We find an interesting oscillation signature existing on large scales of the scale-invariant spectrum and by a detailed simulation we will show this new effect could be detected by the forthcoming astronomical observations, such as PLANCK and LAMOST.

As in inflation theory, our model can be described in terms of scalar fields which minimally couple to the four dimensional Einstein's gravity. Explicitly it consists of two scalar fields ϕ and ψ with the lagrangian given by:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - V(\phi, \psi), \quad (1)$$

in a spatially flat Friedmann-Robertson-Walker (FRW) universe. Here the essential component is the scalar field ψ . It plays a crucial role in giving a bouncing solution smoothly. Without it, the model in Eq. (1) will be similar to the traditional inflation with a single scalar field, which as we know suffers from the problem of the initial singularity. In this model the potential is only the function of the field ϕ and of Coleman-Weinberg form [6]:

$$V = \frac{1}{4}\lambda\phi^4 \left(\ln \frac{|\phi|}{v} - \frac{1}{4} \right) + \frac{1}{16}\lambda v^4, \quad (2)$$

which takes its maximum value $\lambda v^4/16$ at $\phi = 0$ and vanishes at the minima when $\phi = \pm v$. Therefore, the scalar

field ψ merely affects the evolution around the bounce but decays out quickly when away from it.

In order to discuss the perturbations explicitly, we first see how the background universe evolves. In this model a contracting universe can be driven to reach a minimal size during which the universe evolves like a matter-dominant one, and then a quasi-exponential expansion is following, and so is able to explain the problems appeared in standard Big Bang cosmology. The process to link the contraction and expansion is a smooth bounce, and the evolution of the hubble parameter can be treated as a linear function of the cosmic time approximately.

We take the initial condition for the background as that ϕ stays at one vacuum like $-v$ when the universe is contracting and $\dot{\psi}$ is small enough which can be ignored on background evolution. In this phase, the field ϕ oscillates around $-v$ making the equation-of-state (EoS) of the universe oscillate about $w = 0$, and so the average state being similar to a matter-dominated one. Thus we have the useful expressions of background evolution

$$a \sim (-\eta)^2, \quad \mathcal{H} = \frac{2}{\eta}, \quad |\dot{\phi}| \sim \eta^{-3}, \quad (3)$$

where $\mathcal{H} \equiv a'/a$ is the comoving hubble parameter and the prime denotes the derivative with respect to the comoving time. Another useful relation is given by

$$\frac{\phi''}{\phi'} = \frac{2\mathcal{H}\mathcal{H}' - \mathcal{H}''}{2(\mathcal{H}^2 - \mathcal{H}')} , \quad (4)$$

which will be used to calculate the metric perturbations.

Since the universe is contracting, the amplitude of the field ϕ 's oscillation gets larger and larger, while the contribution of the field ψ grows rapidly. When the field reaches the plateau, the bounce happens at the moment t_{B-} . During the bounce, we take the parametrization $H(t) = \alpha(t - t_B)$ around the bounce point t_B , and the coefficient α is a positive constant determined from numerical calculations. In the bouncing phase, the kinetic term of ψ reaches the maximal value and from the equation of motion we deduce an expression $\ddot{\phi}/\dot{\phi} = -3H\dot{\psi}^2/(\dot{\psi}^2 - \frac{\alpha}{4\pi G}) \simeq -3H$ when α is not very

large. Finally we have the approximate relations

$$\mathcal{H} \simeq \frac{y}{2}(\eta - \eta_B), \quad \phi'' \simeq -2\mathcal{H}\phi', \quad |\dot{\phi}| \sim e^{-\frac{3}{4}y(\eta - \eta_B)^2}, \quad (5)$$

where we have defined $y \equiv 8\alpha a_B^2/\pi$.

After the bounce, as the field ϕ moves forward slowly along the plateau, the universe enters into an expanding phase at the moment t_{B+} and the EoS of the universe is approximately -1 . The universe expands with its scale factor growing almost exponentially. In this phase, we have the well-known relations for background evolution

$$a \sim -\frac{1}{\eta}, \quad H \sim \text{Constant}. \quad (6)$$

Finally, when the field drops into the vacuum $+v$, it will oscillate again and the EoS of the universe will oscillate around zero as it does before the bounce. To make the scenario explicit, we give a sketch description of our model in Fig.1 and present the numerical calculation of the background parameters in Fig.2.

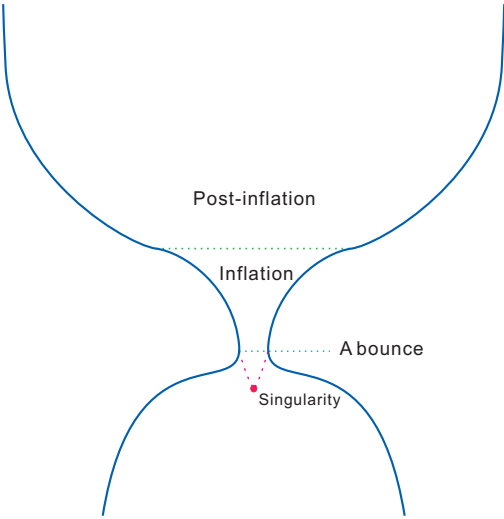


FIG. 1: **Evolution of the universe.** A sketch plot of the evolution of the universe in the model of Eq.(1). Before inflation, there is a bounce instead of the initial singularity.

Now we study the linear perturbations of the model. Taking the longitudinal (conformal Newtonian) gauge, the metric perturbation is presented as follows:

$$ds^2 = a^2(\eta) [(1 + 2\Phi)d\eta^2 - (1 - 2\Psi)dx^i dx^i], \quad (7)$$

and the equation of motion of the gravitational potential is:

$$\begin{aligned} \Phi'' + 2\left(\mathcal{H} - \frac{\phi''}{\phi'}\right)\Phi' + 2\left(\mathcal{H}' - \mathcal{H}\frac{\phi''}{\phi'}\right)\Phi - \nabla^2\Phi \\ = 8\pi G\left(2\mathcal{H} + \frac{\phi''}{\phi'}\right)\psi'\delta\psi, \end{aligned} \quad (8)$$

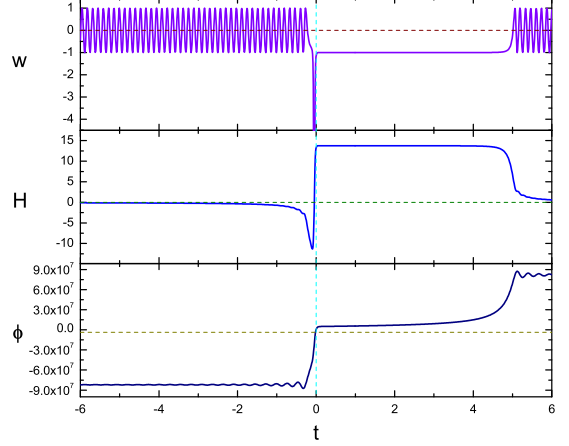


FIG. 2: **Evolutions of ϕ , w and H .** A plot of the evolutions of the field ϕ , the EoS w and the hubble parameter H in the model of Eq.(1) where we take the Coleman-Weinberg potential. In the numerical calculation we choose the parameter $\lambda = 8.0 \times 10^{-14}$, $v = 0.82M_{pl}$, and the initial condition as: $\phi = -0.82M_{pl}$, $\dot{\phi} = 3.0 \times 10^{-10}M_{pl}^2$, $\psi = -0.72M_{pl}$, $\dot{\psi} = 5.0 \times 10^{-13}M_{pl}^2$ where $M_{pl} \equiv 1/\sqrt{G}$.

which can be derived from the basic perturbation equations directly (we refer the complete derivation to Ref.[7], and see e.g. Ref.[8] for a comprehensive survey of the cosmological perturbation theory). As is pointed out previously, the energy density of the field ψ is usually negligible far away from the bounce, and hence we have $\psi' \simeq 0$. Near the bounce, ψ becomes very important, but according to the analysis in Ref.[7] we have the approximation $2\mathcal{H} + \phi''/\phi' \simeq 0$ and so the perturbation of ψ decouples from Eq. (8). Therefore, we will neglect the r.h.s. of Eq. (8), and just focus on the adiabatic fluctuations in the following which can be determined by a single scalar field ϕ .

Now we follow one Fourier mode of the perturbation, labelled by its comoving wave number k , and find that there are two paths for perturbations. The evolution of perturbations is sketched in Fig.3. Initially all the perturbations stay inside the horizon in the far past. Since the hubble radius shrinks, those modes with small comoving wave number exit the horizon while the large k scales still keep inside. When the bounce takes place, all the perturbations will enter the horizon because at that moment the hubble radius diverges. Since in our model the bounce is followed by an expanding phase, those Fourier modes will escape out if the efolds for the post-bounce slow-rolling period is large enough. After that, these modes will re-enter the horizon at late times after the slow-rolling phase has finished. In the following calculations, we will focus on large k region and see whether the large k modes are

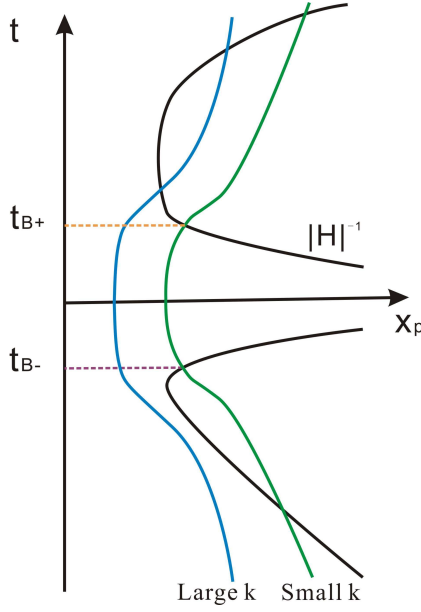


FIG. 3: **Evolution of perturbations.** A sketch plot of the evolution of perturbations with different comoving wave number k in our model.

able to perform scale-invariant spectra and give more information on the CMB observations.

For the contracting phase before the bounce, the equation of motion in momentum space can be solved explicitly. We take the Bunch-Davies vacuum as the initial condition $\Phi_k \sim \frac{4\pi G}{\sqrt{2k^3}} |\dot{\phi}| e^{-ik\eta}$ when the perturbation are deeply inside the horizon. Since during this period $|\dot{\phi}| \sim \eta^{-3}$, we obtain

$$\Phi_k = 4\pi G \frac{\sqrt{\rho_i} \eta_i^3}{\eta^3} \frac{e^{-ik\eta}}{\sqrt{2k^3}}, \quad (9)$$

where the subscript i represents the initial time. Substituting the Eq. (5) into Eq. (8) and solving it, we have the solution to the perturbation in the bouncing phase

$$\Phi_k \simeq e^{-\frac{3}{4}y(\eta-\eta_B)^2} \times \left\{ C_k \cos[k(\eta - \eta_B)] + D_k \sin[k(\eta - \eta_B)] \right\}. \quad (10)$$

Moreover, for the nearly de-Sitter expanding phase, we obtain

$$\Phi_k = (\eta - \tilde{\eta}_{B+})^\gamma [k^{-\nu} E_k J_\nu(k(\eta - \tilde{\eta}_{B+})) + k^\nu F_k J_{-\nu}(k(\eta - \tilde{\eta}_{B+}))], \quad (11)$$

where $\gamma \simeq 1/2$, $\nu \simeq 1/2$ and $\tilde{\eta}_{B+} \equiv \eta_{B+} + 1/\mathcal{H}_{B+}$.

Having obtained the solutions of the perturbation equations in different phases, it is necessary to know the matching relations among these solutions and determine the coefficients C_k , D_k , E_k and F_k . This depends on whether the curvature perturbation on a uniform comoving hypersurface or the gravitational potential passes

through the bounce regularly [9] (see also [10] for a recent study). For a nonsingular bounce scenario such as what we considered, the continuity of background evolution implies that both Φ and Φ' are able to pass through the bounce smoothly. By matching Φ in Eqs. (9) and (10) on the surface η_{B-} , and that in Eqs. (10) and (11) in sub-hubble region on the surface η_{B+} as well as their comoving time derivatives, all of the coefficients can be determined. However, since E_k represents a decaying mode when escape outside the horizon, we neglect it and finally obtain the dominant mode

$$F_k \simeq \sqrt{\frac{\pi}{2}} \frac{4\pi G}{\sqrt{2k^3}} |\dot{\phi}| e^{-ik\tilde{\eta}_{B+}} \times \left\{ 1 + \frac{3e^{-ik(\eta_{B-} - \tilde{\eta}_{B+})}}{k\eta_{B-}} \sin[k(\eta_{B-} - \tilde{\eta}_{B+})] \right\} \quad (12)$$

By comparing the coefficient of Eq. (12) and the initial form of Φ in Eq. (9), one obviously notice that the sub-hubble form of the metric perturbation has obtained an oscillation term when the universe undergoes a bounce. Note that, another important quantity is the curvature perturbation in comoving coordinate $\zeta \equiv \Phi + \frac{\mathcal{H}}{\mathcal{H}^2 - \mathcal{H}'} (\Phi' + \mathcal{H}\Phi)$, and when the expansion is nearly de-sitter like, there is a simple relation between these two quantity $\zeta \simeq \Phi/\epsilon$ with the slow roll parameter $\epsilon \equiv -\dot{H}/H^2$. Therefore, we eventually have the primordial power spectrum for the curvature perturbation

$$P_\zeta \simeq \frac{8}{3} G^2 \frac{\rho}{\epsilon} \left\{ 1 - \frac{3\mathcal{H}_{B-}}{2k} \sin \frac{2k}{\mathcal{H}_{B+}} \right\}, \quad (13)$$

where we have assumed $\eta_{B+} - \eta_{B-}$ to be small compared with $|\frac{1}{\mathcal{H}_{B+}}|$. Obviously, the first term provides a nearly scale-invariant spectrum which is consistent with current cosmological observations. However, the second term apparently shows that there is a wiggle on the spectrum, due to the modification brought by a bounce.

For a numerical estimate of \mathcal{H}_{B+} and \mathcal{H}_{B-} , we normalize the current scale factor $a_0 = 1$ and choose the current hubble parameter $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1} = 1.536 \times 10^{-42} \text{ GeV}$, the hubble parameter during inflation $H_i \simeq 1.68 \times 10^{12} \text{ GeV}$, and the e-folds for inflation $N \simeq 60$. Therefore, our model predicts that $\mathcal{H}_{B+} \simeq 2 \times 10^{-4} \text{ Mpc}^{-1}$ and $\mathcal{H}_{B-} \simeq -1.6 \times 10^{-4} \text{ Mpc}^{-1}$. Based on the primordial spectrum in Eq. (13), in Fig.4 we illustrate the CMB temperature power spectrum and present LSS matter power spectrum. For comparison we have also considered the standard case where \mathcal{H}_{B-} is taken to be zero. In the numerical calculations we use the publicly available Markov Chain Monte Carlo (MCMC) package CosmoMC [11] and take the basic cosmological parameters as given below:

$$(\Omega_b h^2, \Omega_c h^2, \tau, H_0, A_s) = (0.022, 0.115, 0.088, 72, 2.3 \times 10^{-9}), \quad (14)$$

and the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$. One can see from Fig.4 that our model leads to an obvious k -dependent oscillation signature in the power spectrum, especially at large scales.

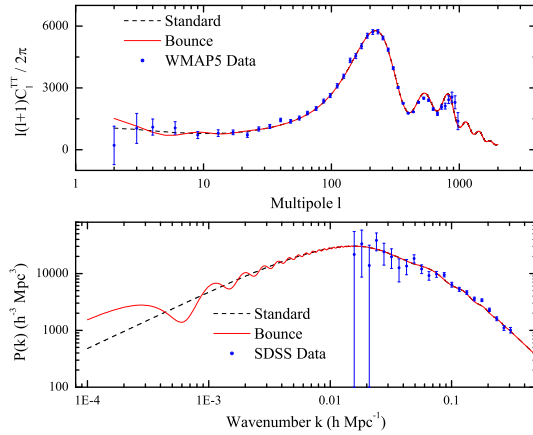


FIG. 4: **Observational effects by a bounce.** The effect on the CMB temperature power spectrum and LSS matter power spectrum by our model $\mathcal{H}_{B+} \simeq 2 \times 10^{-4} \text{ Mpc}^{-1}$ and $\mathcal{H}_{B-} \simeq -1.6 \times 10^{-4} \text{ Mpc}^{-1}$. The dots and error bars are WMAP5 and SDSS data.

To test our model we firstly consider the current astronomical observations from WMAP5 [12] and SDSS [13], due to the large uncertainties at large scales as shown in Fig.4 by the blue dots and the error bars, we find that the oscillating spectrum of our model is consistent with the data. Thus we consider the forthcoming measurements PLANCK [14] and LAMOST [15] with higher precision. We simulate the CMB TT, TE and EE power spectra with the sensitivity of PLANCK and the LSS linear matter power spectrum with the sensitivity of LAMOST and find these measurements will be sensitive to $\mathcal{H}_{B-} \simeq -7.0 \times 10^{-5} \text{ Mpc}^{-1}$ which is smaller than our predicted value and makes our model testable. If this signal would be detected, it will act as a *smoking gun* to the bouncing cosmology.

Physics of bouncing cosmology, since it happens in extremely high energy regime, is hardly to be found by experiments directly. So it is a debate whether a bounce has taken place or not. To find the evidences of a bounce, we need to know what can a bounce leave for observations. This question is still discussed drastically in the literature, and one potential clue is to study the primordial curvature fluctuations. In the context of the Pre-Big-Bang scenario [3] and in the cyclic/Ekpyrotic cosmology [4], the resulting curvature perturbation strongly depends on the physics at the epoch of thermalization, and thus an uncertainty of a thermalized surface is involved [16, 17]. In the frame of loop quantum cosmology,

it is argued that fluctuations before and after the bounce are largely independent [18] (yet see Ref.[19] for some criticisms). In this letter we propose a concrete cosmological model with inflation preceded by a bounce, and by investigating in detail the perturbations we show some imprints of the bounce are detectable to the forthcoming CMB and LSS observations.

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